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ON AN INSTABILITY LINKED WITH THE APPEARANCE OF THE EQUATORIAL SPORADIC E-LAYER

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ON AN INSTABILITY LINKED WITH THE APPEARANCE OF THE EQUATORIAL SPORADIC E-LAYER*

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SUMMARY

The properties are studied of a plasma instability that might be at the origin of the equatorial sporadic E-layer of the ionosphere.

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Farley [1] presented in 1963 an explanation of the appearance of the equatorial sporadic E-layer by the propagation of a plane unsteady wave in the ionospheric plasma in the presence of the equatorial electrojet. This explanation was based upon the solution of the dispersion equation relative to plasma: we indeed obtained from that equation, whose principal characteristics corresponded well to observations, a stable root, that is :

— the instability is produced only if the drift velocity $|\vec{w}_e|$ of electrons in the electrojet is superior to a certain threshold w_0;

— w_0 is of the order of the thermal velocity of ions;

— w_0 increases rapidly when the propagation direction departs from the perpendicular to the terrestrial magnetic field.

However, all these results have been obtained only in a numerical and graphical form; no analytical expressions and, more particularly, no satisfactory physical interpretation of stability has yet been proposed.

This note presents the results of research undertaken with the view of filling these gaps.

To do that, we have considered a plasma constituted of electrons and of a unique kind of positive ions, and we have started from macro-

* Sur une instabilité liée à l'apparition de la couche E sporadique équatoriale.

scopic transport equations comprising for each of the two kinds of particles :

— an equation for the conservation of the number of particles;

— an equation of motion;

— an equation of state, identical to that of a perfect gas.

This last hypothesis implies that the pressure **p** is scalar and isotropic. Besides, we shall assume that it is linked with the density by a law of the form $p \prec \rho^{\gamma}$. Depending upon whether it is imposed for the motions to be isothermic or adiabatic, γ will be taken equal to 1 or 5/3. In the plasma studied this choice is not evident and may differ for the ions and electrons.

This being postulated, the dispersion equation of plane waves of feeble amplitude is obtained as a condition of compatibility of these equations and of the Maxwellian equations. This method differs from that of Farley who utilized for describing the dynamics of particles the linearized Vlasov equations with a type B.G.K - collision term.*

For the low-frequency waves considered, the "longitudinal" approximation, which consists in assuming the electric field of perturbation $\overline{E}_1$ parallel to the direction of propagation, thus neglecting the magnetic field associated with the wave, as perfectly legitimate. The dispersion equation will then be written

$$F_e + F_i + 1 = 0, \tag{1}$$

with

$$F_e = \frac{\pi_e^2}{j\omega_e(j\omega_e + \nu_e)\dfrac{(j\omega_e + \nu_e)^2 + \Omega_e^2}{(j\omega_e + \nu_e)^2 + \Omega_e^2\cos^2\beta} + k^2 u_e^2},$$

$$F_i = \frac{\pi_i^2}{j\omega_i(j\omega_i + \nu_i)\dfrac{(j\omega_i + \nu_i)^2 + \Omega_i^2}{(j\omega_i + \nu_i)^2 + \Omega_i^2\cos^2\beta} + k^2 u_i^2},$$

where :

$\pi_e(\pi_i)$ is the angular frequency of the electron (ion) plasma;

$\Omega_e(\Omega_i)$ is the absolute value of the electron (ion) angular gyrofrequency;

$\nu_e(\nu_i)$ is the collision frequency of electron-neutral (ion-neutral);

$u_e(u_i)$ is the speed of sound characteristic of the thermic agitation of electrons (ions);

$$\omega_e = \omega - \vec{k}.\vec{w}_e; \qquad \omega_i = \omega - \vec{k}.\vec{w}_i;$$

$\vec{w}_e(\vec{w}_i)$ is the drift velocity of electrons (or ions) under the influence

* see add. at the end.

of the static electric field $\vec{E}_0$ associated to the electrojet;

ω is the angular frequency of the wave considered;

$\vec{k}$ is its vector angular wave number; $\hat{k}$ is a unitary vector parallel to $\vec{k}$; k is the module of $\vec{k}\,(\vec{k} = k\hat{k})$;

β is the angle between $\hat{k}$ and the Earth's magnetic field.

In the E-region of the equatorial ionosphere the parameters just examined situate themselves as follows :

$$\begin{cases} \Omega_i \ll \nu_i < \pi_i \sim \nu_e \ll \Omega_e < \pi_e, \\ |\vec{w}_i| < u_i \sim |\vec{w}_e| \ll u_e. \end{cases} \tag{2}$$

Among other consequences, these inequalities point to the fact that the electrons are controlled essentially by the magnetic field, whereas the ions are controlled by neutral molecules by way of collisions.

Exploiting these inequalities it is possible to resolve the equation (1) in an approximate fashion. For wavelengths > 3 m we arrive at the following results :

— the real part of ω is written

$$\mathrm{Re}(\omega) \simeq \vec{k} . \vec{V}_0,$$

with

$$\vec{V}_0 \simeq \frac{\Omega_e \Omega_i}{\Omega_e \Omega_i + \nu_e \nu_i} \vec{w}_e = A \vec{w}_e. \tag{3}$$

In nearly the entirety of the E-region, A is near the unity; that is what is admitted here.

— the imaginary part of ω is written, for $\beta = \pi/2$,

$$\mathrm{Im}(\omega) = \frac{1}{\Omega_e \Omega_i} \left[2u_i^2 - \frac{(\hat{k} . \vec{w}_e)^2}{\left(1 + \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right)^2} \right] k^2 \nu_e. \tag{4}$$

Let us recall that an instability is manifest for a negative value of Im (ω) corresponding to a real value of **k**. This formula thus ascertains quite well the effect of the threshold linked with the situation of the projection of $\vec{w}_e$ over $\hat{k}$ relative to the speed of sound in the ion gas. The threshold w_0 (value of $\hat{k} . \vec{w}_e$ such that Im (ω) = 0), is equal to $\sqrt{2}\, u_i (1 + \nu_i \nu_e / \Omega_i \Omega_e)$.

— w_0 varies with β in the vicinity of $\pi/2$ as follows :

$$w_0 \simeq \sqrt{2}\, u_i \left[1 + \frac{\Omega_e \nu_i}{\Omega_i \nu_e} \left(\frac{\nu_e^2}{\Omega_e^2} + \cos^2\beta \right) \right]. \qquad (5)$$

The ratio ν_e^2/Ω_e^2 is very small, and therefore, formula (5) does imply a vary rapid variation of w_0 as a function of β.

The aggregate of these formulas points, within the limit of approximations made, to a good qualitative and quantitative agreement with the Farley results. This agreement is evidence that within the domain studied the macroscopic approach is satisfactory. Moreover, it emphasizes the kinship, stressed by Dougherty [3], between the simplified B.G.K. term locally conserving the particles utilized in the Vlasov equations, and the collision term which is classical in the magnetoionic theory.

The expression (3) allows another important rapprochement. Indeed, the electrons being controlled by the magnetic field, we have

$$\vec{w}_e \simeq \frac{\vec{E}_0 \times \vec{B}_0}{B_0^2}, \qquad (6)$$

where $\vec{E}_0$ is a static electric field and $\vec{B}_0$ is the induction of the terrestrial magnetic field. As a conseouence,

$$\vec{V}_0 \simeq \frac{\Omega_e \Omega_i}{\Omega_e \Omega_i + \nu_e \nu_i} \frac{\vec{E}_0 \times \vec{B}_0}{B_0^2}. \qquad (7)$$

This expression is no other than that for the drift velocity of weak ionization irregularities in the ionosphere when an electric field, perpendicular to the magnetic field, such as calculated by Clemmow, Johnson [4] and Kato [5] is applied. The first authors, having as here carried out a decomposition into plane waves, have found that the corresponding wave was dampened by diffusion no matter what the drift velocity of electrons. This disagreement stems from the role of ions' inertia which was neglected. It is indeed easy to verify that for weak velocities $|\vec{w}_e|$ their conclusions are again valid. Reciprocally, we find that the inertia of ions plays a determinant part in the instability mechanism; in our opinion, the latter can not be assimilated to the two-beam instability.

This study is part of a research program undertaken under the aegis of the C.N.E.S. (National Center for Space Studies). The results will be published elsewhere at further length.

*** E N D ***

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ADDENDUM to (*) p. 2.

There is question here of a simplified expression of the term proposed by P. L. Bhatnagar, E. P. Gross and M. Krook (Phys.Rev. 511, 1954) to take into account the collisions. To be more precise, one defines a collision frequency independent from the velocity and one writes the collision term of the Boltzmann equation in the form :

$$\left.\frac{\partial n(\vec{r}, \vec{v}, t)}{\partial t}\right|_{\text{coll}} = -\nu n(\vec{r}, \vec{v}, t) + \nu \int n(\vec{r}, \vec{v}, t)\, d^3v\, f_{\text{equilibre}}(v).$$

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